

# Multiplying and Dividing Rational Expressions Notes

## Multiplying and Dividing Rational Expressions

To **multiply** and **divide** two fractions, make use of the following *theorems*:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}, \text{ where } b \neq 0, d \neq 0$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}, \text{ where } b \neq 0, d \neq 0$$

**Sample Problem 1:** Multiply the following rational expressions.

1.  $\frac{8}{y} \times \frac{y^2}{24}$

2.

Solution:

$$\frac{8}{y} \times \frac{y^2}{24} = \frac{8}{\cancel{y}} \times \frac{y(\cancel{y})}{3(\cancel{8})} = \frac{y}{3}$$

2.  $\frac{3x}{x^2-4} \times \frac{x+2}{6}$

Solution:

$$\frac{3x}{x^2-4} \times \frac{x+2}{6} = \frac{3x}{(\cancel{x+2})(x-2)} \times \frac{\cancel{x+2}}{3(2)} = \frac{x}{2(x-2)}$$

3.  $\frac{3y^3}{2x(a+b)} \times \frac{4x^3(a+b)}{ay^3-by^3}$

Solution:

$$\begin{aligned} \frac{3y^3}{2x(a+b)} \times \frac{4x^3(a+b)}{ay^3-by^3} \\ = \frac{3\cancel{y^3}}{2x(a+b)} \times \frac{2(2)(\cancel{x})(x^2)(a+b)}{y^3(a-b)} \\ = \frac{6x^2}{a-b} \end{aligned}$$

4.  $\frac{2x-y}{4x^2-4y^2} \times \frac{x^2+2xy+y^2}{2x^2+xy-y^2}$

Solution:

$$\begin{aligned} \frac{2x-y}{4x^2-4y^2} \times \frac{x^2+2xy+y^2}{2x^2+xy-y^2} \\ = \frac{2x-y}{4(x+y)(x-y)} \times \frac{(x+y)(x+y)}{(2x-y)(x+y)} \\ = \frac{1}{4(x-y)} \end{aligned}$$

**Sample Problem 2:** Divide the following rational expressions.

5.  $\frac{7y}{6x} \div \frac{21y}{36x}$

Solution:

$$\frac{7y}{6x} \div \frac{21y}{36x} = \frac{7y}{6x} \times \frac{36x}{21y} = \frac{7\cancel{y}}{6\cancel{x}} \times \frac{(6)(\cancel{6x})}{3(7\cancel{y})} = \frac{6}{3} = 2$$

7.  $\frac{10ab}{x-3y} \div \frac{-5a^3}{7x-21y}$

Solution:

$$\begin{aligned} \frac{10ab}{x-3y} \div \frac{-5a^3}{7x-21y} &= \frac{10ab}{x-3y} \times \frac{7x-21y}{-5a^3} \\ &= \frac{5a(2b)}{\cancel{x-3y}} \times \frac{7(\cancel{x-3y})}{5a(-a^2)} = -\frac{14b}{a^2} \end{aligned}$$

6.  $\frac{x}{x+5} \div \frac{x}{x+7}$

Solution:

$$\frac{x}{x+5} \div \frac{x}{x+7} = \frac{\cancel{x}}{x+5} \times \frac{x+7}{\cancel{x}} = \frac{x+7}{x+5}$$

8.  $\frac{x^2-9}{x^2-25} \div \frac{x-3}{x+5}$

Solution:

$$\begin{aligned} \frac{x^2-9}{x^2-25} \div \frac{x-3}{x+5} &= \frac{x^2-9}{x^2-25} \times \frac{x+5}{x-3} \\ &= \frac{(x+3)(\cancel{x-3})}{(\cancel{x+5})(x-5)} \times \frac{\cancel{x+5}}{\cancel{x-3}} = \frac{x+3}{x-5} \end{aligned}$$

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**Sample Problem 3:** Perform the indicated operation to the following rational expressions.

9.  $\frac{12x}{x+y} \div \left( \frac{5x-5}{x^2-y^2} \times \frac{3xy}{xy-y} \right)$

Solution:

$$\begin{aligned} & \frac{12x}{x+y} \div \left( \frac{5x-5}{x^2-y^2} \times \frac{3xy}{xy-y} \right) \\ &= \frac{12x}{x+y} \div \left( \frac{5(\cancel{x}-1)}{x^2-y^2} \times \frac{3\cancel{x}y}{\cancel{y}(x-1)} \right) \\ &= \frac{12x}{x+y} \div \frac{15x}{(x^2-y^2)} \\ &= \frac{12x}{x+y} \times \frac{(x^2-y^2)}{15x} \\ &= \frac{\cancel{3}\cancel{x}(4)}{\cancel{x}+\cancel{y}} \times \frac{(\cancel{x}+y)(x-y)}{\cancel{3}\cancel{x}(5)} = \frac{4(x-y)}{5} \end{aligned}$$

10.  $\frac{x^2-25}{x^2-x-12} \div \frac{x^2-x-20}{3x-3} \times \frac{x^2-16}{x^2+4x-5}$

Solution:

$$\begin{aligned} & \frac{x^2-25}{x^2-x-12} \div \frac{x^2-x-20}{3x-3} \times \frac{x^2-16}{x^2+4x-5} \\ &= \frac{x^2-25}{x^2-x-12} \times \frac{3x-3}{x^2-x-20} \times \frac{x^2-16}{x^2+4x-5} \\ &= \frac{(x+5)(x-5)}{(x-4)(x+3)} \times \frac{3(\cancel{x}-1)}{(\cancel{x}-5)(x+4)} \times \frac{(x+4)(x-4)}{(x+5)(x-1)} \\ &= \frac{3}{x+3} \end{aligned}$$